Optimum and Sub-Optimum Linear MMSE Multi-User Detection for Multi-Carrier CDMA Transmission Systems

David Mottier§, Damien Castelain§, Jean-François Hélard*, Jean-Yves Baudais*
§ Mitsubishi Electric ITE – 80, Avenue des Buttes de Coësmes – 35700 Rennes FRANCE
* Laboratoire Composants et Systèmes pour Télécommunications – INSA de Rennes
20, Avenue des Buttes de Coësmes – 35700 Rennes FRANCE
Phone: +33 2 99286685, Fax: +33 2 99386248, e-mail: {mottier, castelain}@tcl.ite.mee.com

Abstract - This paper investigates optimum and sub-optimum linear Minimum Mean Square Error (MMSE) Multi-User Detection (MUD) techniques for multi-carrier code division multiple access transmission systems. In a first step, we present two formulations of MMSE MUD techniques that are optimum for any number of users and any power distribution and achieve exactly the same performance with a different computational complexity. In a second step, we present a simplification of optimum MMSE MUD techniques that achieves sub-optimum performance but at a reduced complexity that is similar to single-user detection techniques. Simulation results on indoor channels show that optimum and sub-optimum MMSE MUD techniques outperform the MMSE single-user detection technique, especially for low and medium system loads as well as in case of near-far effect.

I. INTRODUCTION

In order to obtain multiple access transmission systems with high bandwidth efficiency, Multi-Carrier Code Division Multiple Access (MC-CDMA) combines Orthogonal Frequency Division Multiplex (OFDM) modulation and CDMA [1][2][3]. The OFDM modulation is robust against multipath and ensures good spectral efficiency. The CDMA allows simultaneous communications between different transceivers by allocating to each transmission link a distinct signature (or spreading sequence) that has good orthogonal properties with the other used signatures. Instead of spreading the binary information in the time domain as in the Direct Sequence CDMA technique, the MC-CDMA spreading is performed in the frequency domain. Therefore, the orthogonality among transceivers’ signals has to be ensured in the frequency domain. For point-to-multipoint transmissions, orthogonal signatures such as Walsh-Hadamard sequences are often retained. However, in case of multipath propagation, frequency selective fading destroys this orthogonality and Multiple Access Interference (MAI) occurs, which reduces the system performance. Several detection techniques have been studied to mitigate the MAI degradation. The Single-User Detection (SUD) techniques perform a single tap equalisation per sub-channel of the OFDM multiplex followed by a despreading. They require a low computational complexity and can be based on a simple phase rotation, a zero-forcing equalisation or a Minimum Mean Square Error (MMSE) equalisation, which is known as the most efficient SUD technique [2][5]. However, as SUD-based receivers do not take into account all the characteristics (e.g., the active signatures, the relative power of each signal) of the multiple access transmission, they offer limited performance. Hence, Multi-User Detection (MUD) techniques have been proposed to improve this performance by explicitly considering signals sent to other receivers for the signal detection of one particular receiver. Among the MUD techniques, non-linear solutions based on interference cancellation have been considered in [6][7]. Alternatively, a linear MMSE MUD technique has been described in [2][8]. Its adaptive implementation was developed in [9].

This paper presents new developments on the linear MMSE MUD technique, called "Global MMSE" or G-MMSE, for the forward link of an MC-CDMA transmission system. This detection scheme, which is optimal according to the Mean Square Error (MSE) criterion for any number of active receivers and any power distribution among their signals, has been proposed and evaluated in [10] on non-correlated Rayleigh fading channels and compared to interference cancellation techniques in [11]. After a description of the transmission system in section II, this paper proposes further results in section III and new algorithms on linear MMSE MUD techniques that lead to optimum and sub-optimum performance with a low complexity. The influence of the number of active receivers, the frequency correlation of fading channels as well as the unequal power distribution between signals are considered. Simulation results are presented in section IV for high bit rate indoor MC-CDMA transmission systems based on channel models issued from BRAN HIPERLAN/2 standards [13]. Finally, section V proposes concluding remarks.

II. MC-CDMA SYSTEM DESCRIPTION

As represented on Fig. 1, we consider the forward link MC-CDMA transmission of $K$ independent data streams corresponding to $K$ different users, i.e., mobile stations. For each user $k$ ($k=0,...,K-1$), the data symbol $d^{(n)}(n)$ at time $n$ is multiplied by an energy coefficient $a^{(k)}$ and spread over several subcarriers of the OFDM multiplex, the subcarrier separation being equal to $\Delta_f = 1/T_s$, where $T_s$ is the useful duration of the OFDM symbol. For simplicity reasons, we consider a spreading factor equal to the number of
subcarriers $N_c$ and normalised spreading sequence elements $c_{ij}^{(k)}$ of user $k$ at subcarrier $j$ taking value in $\{-1/N_c, 1/N_c\}$. After OFDM modulation performed by inverse Fast Fourier Transform (FFT) and guard interval insertion, the multi-user OFDM signal is sent through the multipath channel.

At the receiver, after guard interval removal and OFDM demodulation, we get on each subcarrier $r_j(n)$. Assuming that the length of the guard interval is sufficient to absorb the maximum delay spread introduced by the channel, the effect of the channel can be represented by a single complex fading coefficient $h_j(n)$ per subcarrier, which can be considered as constant during each OFDM symbol if the duration of this one is much smaller than the coherence time of the channel. Then, $r_j(n)$ can be written as:

$$r_j(n) = h_j(n) \sum_{k=0}^{K-1} c_{ij}^{(k)} a^{(k)}(n) + n_j(n)$$  

(1)

Denoting matrices (resp. vectors) as upper (resp. lower) case underlined boldface, the received information can also be written as:

$$r(n) = [r_0(n), \ldots, r_{N_c-1}(n)]^T = \mathbf{H}(n) \mathbf{Cd}(n) + \mathbf{n}(n)$$  

(2)

where $\mathbf{H}(n) = \text{diag}[h_0(n), \ldots, h_{N_c-1}(n)]$ is a $N_c \times N_c$ diagonal matrix containing the complex channel attenuation of each subcarrier, $\mathbf{C} = [c_{00}^{(0)}, \ldots, c_{K-1}^{(K-1)}]$ is a $N_c \times K$ matrix, the columns of which are the spreading sequences of each active user, $\mathbf{A} = \text{diag}[a_0^{(0)}, \ldots, a_{K-1}^{(K-1)}]$ is a $K \times K$ diagonal matrix containing the energy coefficient associated to each user, $\mathbf{d}(n) = [d_0^{(0)}(n), \ldots, d_{K-1}^{(K-1)}(n)]^T$ is a $K \times 1$ vector containing the data symbols of each user and $\mathbf{n}(n)$ is a $N_c \times 1$ vector of Additive White Gaussian Noise (AWGN) components.

### III. LINEAR MMSE MULTI-USER DETECTION

In order to mitigate the loss of orthogonality between the signals of different users, we may propose several types of MMSE detectors.

#### A. Optimum Linear MMSE Multi-User Detection

A first technique, called Global-MMSE or G-MMSE, relies on the a priori knowledge of the spreading sequences and the signal power of each active user [10]. This technique expresses the results of all users in one step and in matrix formulation. Applying the MSE criterion to received vector $\mathbf{r}(n)$, the G-MMSE detector yields the vector of decision variable $\mathbf{y}(n)$ as:

$$\mathbf{y}(n) = \mathbf{W}^H(n) \mathbf{r}(n)$$  

where

$$\mathbf{W}(n) = \Gamma^{-1}_{\mathbf{r}\mathbf{r}}(n) \cdot \Gamma^{-1}_{\mathbf{r}\mathbf{d}}(n)$$  

(3)

And

$$\Gamma_{\mathbf{r}\mathbf{r}}(n) = E \mathbf{r}(n) \mathbf{r}^H(n) = \mathbf{H}(n) \mathbf{C} \mathbf{A}^2 \mathbf{H}^* (n) + \sigma^2 \mathbf{I}_{N_c}$$  

(4)

$$\Gamma_{\mathbf{r}\mathbf{d}}(n) = E \mathbf{r}(n) \mathbf{d}^H(n) = \mathbf{H}(n) \mathbf{C} \mathbf{A}$$  

(5)

where $\mathbf{I}_{N_c}$ is the $N_c \times N_c$ identity matrix. Thus, vector $\mathbf{y}(n)$ can be written as follows:

$$\mathbf{y}(n) = \mathbf{AC}^H \mathbf{H}^* (n) \left( \mathbf{H}(n) \mathbf{C} \mathbf{A}^2 \mathbf{H}^* (n) + \sigma^2 \mathbf{I}_{N_c} \right)^{-1} \mathbf{r}(n)$$  

(6)

which can be interpreted as a first process of equalisation with matrix $\mathbf{H}^* (n) \mathbf{H}(n) \mathbf{C} \mathbf{A}^2 \mathbf{H}^* (n) + \sigma^2 \mathbf{I}_{N_c}$ followed by a second process of despreading with matrix $\mathbf{AC}^H$. Rewriting this to generate results for a specific user only, we would get an expression equivalent to the one proposed in [2]. This detection technique achieves excellent performance, namely for low to medium system loads, i.e., when the number of active users is low or medium compared to the maximum number of possible users, and for users with different power levels. However, whatever the number of active users $K$, this detection implies to solve a $N \times N$ linear system, which represents a large complexity.

Therefore, we consider an alternative formulation, which is completely equivalent and still optimum according to the MSE criterion. Similarly to a DS-CDMA MMSE receiver which relies on a preliminary Rake processing before MMSE filtering, we first apply the matched filter to received vector $\mathbf{r}(n)$. This first filtering process can be interpreted as a maximum ratio combining (MRC). It yields a $K \times 1$ signal vector $\mathbf{z}(n)$, which is an exhaustive summary of $\mathbf{r}(n)$ defined as:

$$\mathbf{z}(n) = [z_0(n), \ldots, z_{K-1}(n)]^T = \mathbf{AC}^H \mathbf{H}^* (n) \mathbf{r}(n)$$  

(7)

We then apply the MSE criterion to vector $\mathbf{z}(n)$ and similarly to equation (3), we get the vector of decision variable $\mathbf{y}(n)$ as:

$$\mathbf{y}(n) = \mathbf{W}^H(\mathbf{z}(n)) \mathbf{z}(n)$$  

with

$$\mathbf{W}(\mathbf{z}(n)) = \Gamma^{-1}_{\mathbf{z}\mathbf{z}^*}(\mathbf{z}(n)) \cdot \Gamma^{-1}_{\mathbf{z}\mathbf{d}}(\mathbf{z}(n))$$  

(8)
Using this new formulation, we get a new expression for vector $y(n)$:

$$y(n) = \left( \mathbf{A}^{H} \mathbf{H}^{2}(n) \mathbf{C} \mathbf{A} + \sigma^{2} \mathbf{I}_{K} \right)^{-1} \mathbf{A}^{H} \mathbf{H}^{i}(n) \mathbf{r}(n)$$  \hspace{1cm} (11)

Equations (6) and (11) achieve exactly the same performance and thus will both be referred as G-MMSE in the sequel. However, by contrast with equation (6), applying equation (11) only implies to solve a $K \times K$ linear system. In other words, equation (11) has a lower complexity than equation (6) for any but full system load. For full system load and if users have the same energy ($a^{i}=a$), solving (6) is greatly simplified since the matrix that has to be inverted becomes diagonal. Thus, before despreading, a single equalisation coefficient is applied on each sub-channel and the decision variable of user $i$ becomes:

$$y^{i}(n) = \frac{N_{c}-1}{\bar{h}^{2}(n)} \sum_{k=0}^{N_{c}} a^{i} \cdot h^{*}_{j}(n) \cdot c_{j}^{(i)*} \cdot r_{j}(n)$$  \hspace{1cm} (12)

As mobile stations have drastic computational restrictions due to power consumption limitations, the implementation complexity of both linear G-MMSE relations based on solving a linear system may not be tolerable even for low system loads or low spreading factors. Therefore, we propose a simplification of the G-MMSE technique that avoids solving a linear system

**B. Sub-Optimum Linear MMSE Multi-User Detection**

As a first simplification, we recall an expression of the MMSE SUD technique, also called Per Carrier MMSE [2] or PC-MMSE, which relies on single tap equalisation per subcarrier and can be viewed as a generalisation of equation (12) for any system load ($0 < K \leq N_{c}$) and any difference between the energies of user’s signals:

$$y^{i}(n) = \sum_{k=0}^{N_{c}} a^{i} \cdot h^{*}_{j}(n) \cdot c_{j}^{(i)*} \cdot r_{j}(n)$$  \hspace{1cm} (13)

This technique belongs to SUD methods since no knowledge about the spreading sequences of other users is required to detect the signal of the desired user. It can also be interpreted as a simplification of (6) where matrix $\mathbf{H}^{2}(n) \mathbf{C} \mathbf{A} \mathbf{A}^{H} \mathbf{H}^{i}(n)$ is approximated to its diagonal. For full system load and if users’ signals have equal power, PC-MMSE and G-MMSE achieve exactly the same performance. However, PC-MMSE is sub-optimal in any other cases since the matrix terms outside the diagonal that take into account the channel frequency correlation and power differences are neglected. Hence, we may propose an alternative simplification of the G-MMSE to improve the performance of the transmission as compared with the PC-MMSE while keeping the same level of complexity for its implementation in mobile stations. At first, we rewrite (13) as:

$$y^{i}(n) = \frac{N_{c}-1}{\bar{h}^{2}(n)} \sum_{k=0}^{N_{c}} a^{i} \cdot h^{*}_{j}(n) \cdot c_{j}^{(i)*} \cdot r_{j}(n)$$  \hspace{1cm} (14)

where $I_{k}^{i}(\ell) = \sum_{k \neq 0, k \neq \ell} a^{k} \cdot |r_{j}(n)|^2$ is a chip-level interference power coming from the existence of all active users except user $i$ on each component of the non-equalised received vector $\mathbf{r}(n)$. The proposed solution consists in still rewriting (14) as a function of a symbol-level interference $I_{k}^{i}(\ell)$, which is also computed from the non-equalised signal but after despreading.

Let us first evaluate the average power of decision variable $y^{i}(n)$ computed when no equalisation is performed. We get three distinct contributions:

$$E \left[ |y^{i}(n)|^{2} \right] \bigg|_{N_{c} \text{ Equs.}} = E \left[ \sum_{\ell=0}^{N_{c}-1} c_{j}^{(i)*} r_{j}(n) \right]^{2} = a^{i} \cdot \sum_{\ell=0}^{N_{c}-1} \sum_{m=0}^{N_{c}-1} c_{j}^{(i)*} c_{m}^{(j)*} E \left[ h_{j}(n) h_{m}(n) \right] + \sigma^{2} \cdot \frac{N_{c}}{N_{c}}$$  \hspace{1cm} (15)

where $w_{j}^{(i)} \cdot c_{j}^{(i)*} c_{j}^{(j)*}$ is the product between the elements of the $i$-th and $\ell$-th spreading sequence at subcarrier $\ell$.

The first part of equation (15) is the influence of the desired signal, the second part is due to AWGN whereas the third part results from interfering users and is what we call symbol-level interference $I_{\ell}^{i}(\ell)$.

For channels that are frequency non-correlated, we get $E[h(n)h^{*}(n)]=0$ for $\neq m$, so that equation (15) becomes:

$$E \left[ |y^{i}(n)|^{2} \right] \bigg|_{N_{c} \text{ Equs.}} = a^{i} \cdot \sum_{\ell=0}^{N_{c}-1} \sum_{m=0}^{N_{c}-1} c_{j}^{(i)*} E \left[ h_{j}(n) \right]^{2} + \sigma^{2} \cdot \frac{N_{c}}{N_{c}}$$  \hspace{1cm} (16)

Considering the $1/N_{c}$ magnitude of each spreading sequence element and assuming that the channel lets the overall signal power unchanged, we get:
\[ E\left[ h_i(n)^2 \right] = 1 \]  
(17)

and

\[
E\left[ y(i)(n) \right] = \frac{a(i)^2}{N_C} + \frac{\sigma^2}{N_C} + \frac{1}{N_c} \sum_{k=1}^{K-1} a(k)^2
\]

(18)

where the last term, which is the symbol-level interference, is equal to the chip-level interference divided by the interference reduction factor \( N_c \). Therefore, we rewrite equation (14) as:

\[
y(i)(n) = \sum_{j=0}^{N_c-1} a(i) \cdot h_j(n) \cdot c_j \cdot r_j(n)
\]

(19)

where:

\[
I_S = \sum_{k=1}^{K-1} a(k)^2 \sum_{l=0}^{N_c-1} w_l(k) \sum_{m=0}^{N_c-1} w_m^*(k) \cdot E\left[ h_i(n) h_m^*(n) \right]
\]

(20)

As equivalence between (14) and (19) has been demonstrated for frequency non-correlated fading channels, both solutions will experience the same performance for such transmission conditions. However, we expect to obtain better performance by extending the validity of (19) to the context of frequency correlated fading channels. Indeed, it was shown in [12] that the frequency channel correlation itself could reduce or increase the multiple access interference depending on the selected subset of spreading sequence. Thus, designing an MMSE detection scheme on an apparent chip-level interference as in (14) may lead to sub-optimal performance compared with an MMSE detection scheme based on the true remaining interference computed after despreading as in (19).

As the computation of equation (20) still relies on the a priori knowledge of the spreading sequences and the signal power of each active user, this new linear sub-optimum G-MMSE detector, i.e., SG-MMSE, belongs to MUD techniques. Compared to G-MMSE, it avoids solving a linear system.

**IV. SIMULATION RESULTS**

In order to compare the performance of G-MMSE, PC-MMSE and SG-MMSE detection techniques, we consider a downlink synchronous MC-CDMA transmission scheme using 64FFT-based OFDM modulation and Walsh-Hadamard spreading sequences of length 64. The signal bandwidth is 20 MHz and perfect power control is assumed, i.e., the instantaneous power of the channel impulse response is unity. The performance of this transmission is evaluated in terms of bit error rate (BER) for two Rayleigh fading indoor propagation channels, which are issued from the HIPERLAN/2 ETSI-BRAN standard [13], with a distinct coherence bandwidth \( B_c \); \( B_c = 2.56 \) MHz for channel A whereas \( B_c = 1.2 \) MHz for channel B.

On Fig. 2 is represented the influence of the channel frequency correlation on the BER for the transmission of 10 active users with equal power signals using the three different MMSE detection techniques. The BER is averaged over all the users. As expected, for any channel frequency correlation, G-MMSE outperforms other detection techniques whereas PC-MMSE detection performs worst. To achieve a BER of \( 10^{-3} \) on channel A, SG-MMSE only experiences a 0.3 dB loss compared to G-MMSE. Its gain is then 0.7 dB compared to PC-MMSE. This gain is reduced for channel B. In this latter case, the channel frequency correlation is lower and does not significantly improve the MAI mitigation capability so that considering MAI before or after despreading in a one-tap-per-carrier equalisation structure gives very similar performance.

![Fig. 2: Influence of channel frequency correlation on the performance of G-MMSE, SG-MMSE and PC-MMSE detection techniques (10 users, Channels A and B).](image)

Fig. 3 depicts the influence of system load on the average BER of a transmission over channel A for the three MMSE detection techniques.

![Fig. 3: Influence of system load on the performance of G-MMSE, SG-MMSE and PC-MMSE detection techniques (Channel A,)](image)
Each user has the same signal-to-noise ratio $Eb/N0 = 8$ dB. As expected, G-MMSE, SG-MMSE and PC-MMSE achieve exactly the same performance for a number $K$ of active users equal to 64, which is full load. In any other case, G-MMSE outperforms other techniques with a significant performance improvement for low to medium loads. For loads up to 16 users, G-MMSE achieves near single-user performance. This excellent result comes from the selected subset of active spreading sequence, which is optimised and induces a high benefit from the channel frequency correlation in the MAI mitigation process [12]. For higher loads, the degree of freedom in the selection of spreading sequences among the 64 available is no more sufficient to avoid “bad” spreading sequences in the optimised subset. Then, MAI grows with the number of users. Equivalently, for loads up to 16 users, SG-MMSE experiences better performance than PC-MMSE since taking the symbol-level interference into account in (19) gives more benefit from the MAI mitigation process.

G-MMSE achieves the best performance, namely for low to medium system loads or if users’ signals have a different power. However, even in the alternative formulation proposed in this paper, G-MMSE may lead to excessive computational complexity. By contrast, PC-MMSE has a low complexity but experiences significant performance degradations. As a good performance–complexity trade-off, we have proposed SG-MMSE, which derives from G-MMSE, and achieves better performance than PC-MMSE. SG-MMSE is all the more efficient as the channel is frequency correlated and is more robust against near-far effect.

**REFERENCES**


