Pilot-based CSI Feedback in TDD/MIMO Systems with Cochannel Interference

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Abstract—This paper proposes a pilot-based channel state information (CSI) feedback in time-division duplex (TDD) / multi-input multi-output (MIMO) system, which enables the transmitter to perform transmit beamforming considering the receiver’s interference. In the proposed method, the receiver sends reverse-link pilot signals spatially prefiltered by linear precoder dependent on the receiver’s interference characteristics. The transmitter exploits responses of the reverse-link pilot signals to perform transmit beamforming considering the receiver’s interference. The transmitter also predicts the receiver’s output signal-to-interference-plus-noise power ratio (SINR) for each data stream. From simulation results, it is found that the transmitter achieves efficient transmission performance maintaining small control errors based on the proposed CSI feedback.

I. INTRODUCTION

In recent years, efficient transmission schemes in space and frequency domains have been intensively investigated to meet the growing demand for high data rate wireless communications. Specifically in space domain, multi-input multi-output (MIMO) systems with multiple antennas at both transmitter and receiver have gained much attention to exploit the potential increase in spectral efficiency.

Many transmission schemes in MIMO channel have been studied mainly from two different approaches. One approach is the case of no channel knowledge at the transmitter, where parallel data streams are transmitted from individual transmit antennas [1]–[4]. Although this transmission scheme is simple, the performance greatly depends on the channel conditions. Another approach is to use channel knowledge at the transmitter [5]–[7], where parallel data streams are transmitted from different transmit beamformers. Using appropriate transmit beamforming, this approach is expected to achieve more efficient transmission than the first approach.

In practice, the channel state information (CSI) feedback is essential to realize the appropriate transmit beamforming at the transmitter. In reciprocal time-division duplex (TDD) systems where the channel responses between forward and reverse links are reciprocal, the transmitter can obtain MIMO channel gains, measuring channel responses of reverse-link pilot signals individually transmitted from the receiver’s antennas with equal power. This generic channel measurement would be effective if the receiver has only white noise component. However, when the receiver has spatially colored cochannel interference, the transmitter cannot take into account the receiver’s interference effect in performing transmit beamforming. As a result, the performance of MIMO system will deteriorate, because the transmitted data stream might have the similar arrival angle to the co-channel interference at the receiver.

Theoretically, this performance deterioration can be avoided by transmit beamforming which takes into account the receiver’s interference, if the interference characteristics are known at the transmitter [7][8]. However, it is impractical to feed back huge information bits of interference characteristics through reverse-link. Therefore, a feasible CSI feedback including the receiver’s interference is required to perform the transmit beamforming, although it has not been proposed yet.

In this paper, we propose a new pilot-based CSI feedback method, which enables the transmitter to perform transmit beamforming considering the receiver’s interference effect. In the proposed method, the receiver sends reverse-link pilot signals spatially prefiltered by linear precoder dependent on the receiver’s interference characteristics. The transmitter measures joint responses of MIMO channel and the linear precoder through reverse-link, and exploits the responses for transmit beamforming. Accordingly, the transmitter can consider the receiver’s interference effect in transmit beamforming, keeping the same amount of reverse-link signalling to the generic pilot signalling for channel measurement. The transmitter also predicts the receiver’s output signal-to-interference-plus-noise power ratio (SINR) corresponding to each transmit beamformer and selects available transmit beamformers for data transmission. In ideal conditions, the transmit beamforming in our control scheme corresponds to one which can maximize channel capacity of MIMO system. We demonstrate that the proposed method enables efficient transmission performance in realistic environments, maintaining small control errors.

II. MIMO SYSTEM WITH TRANSMIT BEAMFORMING

Throughout the paper, we define the transpose as $^T$, the complex conjugate as $^*$, the complex conjugate transpose as $^H$, the norm as $|| \cdot ||$, and the trace as $\text{tr} \{ \cdot \}$.

Consider MIMO system which is composed of a transmitter (hereafter, “terminal A”) with $N$ antennas and a receiver (hereafter, “terminal B”) with $M$ antennas. The channel is assumed quasi-stationary flat fading, which is a typical environment of low-mobility terminals. The MIMO channel is denoted as a $M \times N$ matrix $H$, whose $(m, n)$-th element is the complex
propagation gain from the terminal A's n-th antenna to the terminal B's m-th antenna.

Figure 1 shows the block diagram of MIMO system with transmit beamforming. The terminal A sends \( n_{\text{max}} \leq N \) data streams simultaneously using different transmit beamforming based on the \( N \times 1 \) weight vectors \( w_n \) (\( ||w_n|| = 1 \), \( n = 1, ..., n_{\text{max}} \)). Assuming that the n-th transmit beamformer sends signal \( s_n(p) \) (\( E[|s_n(p)|^2] = 1 \)) with power \( P_S^{(n)} \), the \( M \times 1 \) received signal \( x(p) \) at the terminal B corresponding to the p-th sample is given by

\[
x(p) = \sum_{n=1}^{n_{\text{max}}} H w_n \sqrt{P_S^{(n)}} s_n(p) + z_{IN}(p)
\]

(1)

where \( z_{IN}(p) \) is the \( M \times 1 \) interference-plus-noise vector at terminal B with \( E[|z_{IN}(p)|^2] = R_{IN} \). Using the \( M \times 1 \) receive weight vector \( v_n \), for the n1-th data stream, the terminal B obtains the n1-th output \( y_{n1}(p) \) as

\[
y_{n1}(p) = \sum_{n=1}^{n_{\text{max}}} v_{n1}^T H w_n \sqrt{P_S^{(n)}} s_n(p) + v_{n1}^T z_{IN}(p).
\]

(2)

If the n-th transmit weight \( w_n \) (\( n = 1, ..., n_{\text{max}} \)) satisfies

\[
H^H R_{IN}^{-1} H w_n = \rho_n w_n
\]

(3)

with the n-th largest eigenvalue \( \rho_n \) of \( H^H R_{IN}^{-1} H \) and if the n-th receive weight \( v_n \) is given by \( v_n = (R_{IN} H w_n)^* \), we have [8]

\[
v_{n1}^T H w_{n2} = \begin{cases} \rho_n, & n_1 = n_2 \\ 0, & \text{otherwise} \end{cases}
\]

(4)

Thus, MIMO channel is decoupled by the above combinations of transmit weight \( w_n \) and receive weight \( v_n \). Since the signal power and interference-plus-noise power at the output \( y_{n1}(p) \) are \( P_S^{(n)} \rho_n \), and \( E[|w_{n1}^T z_{IN}(p)|^2] = \rho_n \), respectively, the receiver's output SINR is given by \( P_S^{(n)} \rho_n \). Theoretically, the above set of transmit and receive weights is known as a solution which can maximize the channel capacity in MIMO channel [7][8].

To realize the transmit beamforming (3), the terminal A needs knowledge of \( H \) and \( R_{IN} \). However, it is impractical that the terminal B feeds back huge information bits of \( R_{IN} \) to the terminal A. In the next section, we propose a feasible CSI feedback method, which enables the terminal A to perform the transmit beamforming (3).

### III. Reverse-Link Signalling for Transmit Control

We propose a new pilot-based CSI feedback from the terminal B to the terminal A to achieve the transmit beamforming (3) in reciprocal TDD/MIMO systems.

#### A. Reverse-Link Pilot Signalling

Assume that the terminal B has knowledge of correlation matrix \( R_{IN} \). The matrix \( R_{IN} \) can be usually estimated by averaging \( z_{IN}(p) z_{IN}^H(p) \) over a large number of samples at the terminal B. In the proposed method, the terminal B sends \( M \) parallel pilot signals spatially prefiltered by \( M \times M \) linear precoding matrix \( \sqrt{\eta} G \), where \( \eta \) is a representative parameter of the pilot transmission power. The matrix \( G \) is determined as

\[
G = F^* \Phi^{-1/2}
\]

(5)

where \( \Phi \) and \( F \) are the \( M \times M \) diagonal and unitary matrices, respectively, obtained from eigen decomposition of \( R_{IN} = F \Phi F^* \).

The pilot signals \( r_1(p), ..., r_M(p) \) have \( p_b \) symbols (\( p_b = 1, ..., p_b \)) and are mutually orthogonal as

\[
\frac{1}{p_b} \sum_{p=1}^{p_b} r_{m1}(p) r_{m2}^*(p) = \begin{cases} 1, & m_1 = m_2 \\ 0, & \text{otherwise} \end{cases}
\]

(6)

Along with the pilot transmission, the terminal B feeds back information to the terminal A.

In reciprocal TDD systems, the \( p \)-th sample \( x_A(p) \) of the terminal A’s received signal is expressed as

\[
x_A(p) = \sqrt{\eta} H^T G r(p) + z_A(p)
\]

(7)

\[
r(p) = [r_1(p), ..., r_M(p)]^T
\]

(8)

where \( z_A(p) \) is the \( N \times 1 \) interference-plus-noise vector at the terminal A. The received signals \( p = 1, ..., p_b \) are totally expressed in a matrix form as

\[
X_A = [x_A(1), ..., x_A(p_b)] = \sqrt{\eta} H^T G R + Z_A
\]

(9)

\[
R = [r(1), ..., r(p_b)]
\]

\[
Z_A = [z_A(1), ..., z_A(p_b)]
\]

where \( RR^H / p_b = I \) from (6).

#### B. Transmit Beamforming and SINR Prediction

Using the feedback information \( \eta \), the terminal A estimates \( H^T G \) as

\[
J = \frac{1}{\sqrt{\eta p_b}} X_A R^H = H^T G + \frac{1}{\sqrt{\eta p_b}} Z_A R^H
\]

(10)

In case of \( Z_A = 0 \), it holds that

\[
J^* J^T = H^T G^* G^T H = H^T R_{IN}^{-1} H
\]

(11)

Therefore, the terminal A determines the transmit weight \( w_n \) and predicts the terminal B’s received SINR \( \gamma_n^{(p)} \) as

\[
w_n = e_n (J^* J^T)
\]

(12)

\[
\gamma_n^{(p)} = P_S^{(n)} \cdot \rho_n (J^* J^T)
\]

(13)
where \( \lambda_n \) and \( e_n \) are the \( n \)-th largest eigenvalue and the corresponding eigenvector of the matrix \( \langle \cdot \rangle \), respectively. In case of \( Z_A = 0 \), the transmit weight \( w_n \) and the predicted SINR \( \gamma_{p[r]} \) satisfy the ideal condition (3). Thus, the terminal A can perform transmit beamforming considering the terminal B’s interference effect without individual knowledge of \( H \) and \( R_{IN} \).

In the proposed method, the reverse-link pilot signals are linearly precoded depending on the terminal B’s interference and the terminal A can reflect the terminal B’s interference on transmission control. The terminal A always uses the same control procedures independently of the terminal B’s interference. Nevertheless, the terminal A can consider the terminal B’s interference effect.

In a special case of no interference at the terminal B, \( R_{IN} \) corresponds to \( P_N I \), where \( P_N \) is the noise power per antenna. Then, the terminal B sends parallel pilot signals using \( \sqrt{\eta G} = \sqrt{\eta P_N I} \). If \( Z_A = 0 \), (11) is reduced to

\[
J^* J^T = \frac{1}{P_N} H^4 H. \quad (14)
\]

The condition (14) gives transmit weights to maximize MIMO channel capacity under noise environments [5].

C. Discussions

To understand principle of the proposed method, let us consider the matrix \( D = (F^* \Phi^{-1/2})^T \). The matrix \( D \) corresponds to a whitening matrix in forward-link, which whitens the terminal B’s interference-plus-noise \( z_{IN}(p) \) as

\[
E[z_{IN}(p) z_{IN}(p)^*] = D R_{IN} D^T = I \quad \text{with} \quad z_{IN}(p) = Dz_{IN}(p). \]

Hence, the received signal \( x(p) \) after the whitening process is expressed as

\[
x(p)^* = \sum_{n=1}^{N} D H w_n \sqrt{P_n^T} z_n(p) + z_{IN}(p). \quad (15)
\]

Figure 2 shows the equivalent block diagrams of the terminal B. Since \( H^T R_{IN} H = (DH)^T (DH) \), the transmit beamforming of (3) corresponds to eigenbeamforming under the virtual channel \( DH \) and white interference-plus-noise \( z_{IN}(p) \) [8], as shown in figure 2(a). Therefore, the terminal A can yield transmit weights of \( (3) \) if the virtual channel \( DH \) is obtained.

The proposed method applies this principle and the terminal B equivalently sends the pilot signals \( \sqrt{\eta r}(p) \) from the end of \( D \) in figure 2(b), so that the terminal A can estimate the virtual channel \( DH \). Then, actual signals transmitted from the terminal B are expressed as \( \sqrt{\eta D^T r(p)} \). Since the interference-plus-noise component is spatially white in the virtual channel, it is not necessary to feed back the correlation coefficients of interference-plus-noise between the terminal B’s antennas. Consequently, the proposed method feeds back only a single parameter \( \eta \) and does not require a large amount of feedback information.

Although we presented a case of single-user MIMO systems, the proposed CSI feedback could be applied to multiuser MIMO systems based on the principle, in which each terminal feeds back the pilot-based CSI to the base station. Thus, the proposed scheme has potential advantages in cellular mobile communication systems with co-channel interference.

IV. Numerical Results

The proposed method is evaluated by computer simulations.

A. Simulation Parameters

The simulation parameters are listed in Table I. In the simulations, a quasi-static Rayleigh fading channel is assumed, in which elements of the channel matrix \( H \) are independent identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance. We assume that all control procedures are performed within coherence time of fading channel in a simulation trial and that simulation trials have independent fading channels.

Terminal B has interference-plus-noise \( z_{IN}(p) = \sqrt{\eta_I} h^{(i)} \cdot i(p) + z_N(p) \), where \( i(p) \) (\( E[|i(p)|^2] = 1 \)) is the interference signal with unit power, \( P_I \) is the interference power, \( z_N(p) \) is the noise vector with power \( P_N \) per antenna, and \( h^{(i)} \) is the interference response vector, elements of which have i.i.d. complex Gaussian random variables with zero mean and unit variance. The response vector \( h^{(i)} \) is assumed as constant in a simulation trial and independent between different trials. Terminal A has only noise component with power \( P_N \) per antenna.

In the simulations, the terminal B initially estimates correlation matrix \( \tilde{R}_{IN} \) using \( p_{IN} = 16 \) samples as \( \tilde{R}_{IN} = (1/p_{IN}) \sum_{n=p_{IN}-1}^{n=p_{IN}-1} z_{IN}(p) z_{IN}(p)^* \). Next, the terminal B determines the \( M \times M \) precoding matrix \( \sqrt{\eta G} \) using \( \tilde{R}_{IN} \) and sends binary phase-shift keying (BPSK) orthogonal pilot signals composed of \( p_0 = 8 \) symbols. Assuming that the terminal B has the average transmit power \( P_S \) per antenna, we have

\[
P_S = \frac{\eta}{M} \|G r(p)\|_2^2 = \frac{\eta}{M} \text{tr}(G G^*) = \frac{\eta}{M} \text{tr}(\tilde{R}_{IN}^{-1}).
\]

Therefore, the terminal B determines \( \eta \) as \( M P_S \text{tr}(\tilde{R}_{IN}^{-1})^{-1} \).

According to (12) and (13), the terminal A computes transmit weights \( w_n \) and predicts the SINRs \( \gamma_{p[r]} = P_S \rho_n \).
for \( n = 1, \ldots, \min\{N, M\} \), supposing constant transmit power for each beamformer \( P_S^{(n)} = P_S \). Furthermore, the terminal A selects transmit beamformers to meet the required SINR and sends the data stream using the transmit weight \( w_n \).

In the beginning of data stream, BPSK orthogonal pilot signals with \( p_1 = 8 \) symbols are transmitted and followed by data packets with QPSK modulation. Each data packet includes 432 information bits which are convolutionally coded by \( K = 7 \) and \( r = 3/4 \). From packet error rate (PER) results in SISO channel with additive white Gaussian noise, the required SNR to meet \( \text{PER}=10^{-3} \) is given by 6.0[dB]. Accordingly, transmit beamformers with predicted SINR beyond 6.0 [dB] are selected. If the predicted SINR is below 6.0[dB], the transmit beamformer is not used and, then, the total transmit power becomes less than \( \min\{N, M\} \cdot P_S \).

In actual environments, each terminal has phase errors in analog circuits between forward and reverse links, due to different RF circuit lengths and amplifier’s characteristics. Although the phase errors can be reduced by the use of calibration, a small phase error still remains and reciprocal channel responses are not perfect. In the presence of phase errors, the transmit weights in the terminals A & B are equivalently changed as \( w_n \rightarrow \phi_A w_n \) with \( \phi_A = \text{diag}[e^{j\phi_{A,1}}, \ldots , e^{j\phi_{A,N}}] \) and \( G \rightarrow \phi_B G \) with \( \phi_B = \text{diag}[e^{j\phi_{B,1}}, \ldots , e^{j\phi_{B,N}}] \), respectively. Here, \( \phi_{A,1}, \ldots , \phi_{A,N}, \phi_{B,1}, \ldots , \phi_{B,M} \) are the phase errors of independent Gaussian random variables with standard deviation \( \Delta_\phi = 0.03\pi \).

To receive the \( n \)-th data stream, the terminal B computes the MMSE weight \( \tau_n \) using pilot signals \( s_n(p) \) with \( p_1 = 8 \) symbols. We define SINR prediction error \( \sigma_n^{(p)} \) between actual output SINR \( \gamma_n \) and predicted SINR \( \gamma_n^{(p)} \) in the \( n \)-th beamformer as

\[
\sigma_n^{(p)} = \sqrt{\mathbb{E}\left[ (10 \log_{10} \gamma_n - 10 \log_{10} \gamma_n^{(p)})^2 \right]}
\]

where \( \mathbb{E}\{\cdot\} \) denotes the average over 50,000 simulation trials.

### B. Benchmark

For comparison purpose, we consider the conventional method, in which the terminal B sends reverse-link pilot signals without taking into account the spatially colored interference. In the conventional method, the terminal B sends pilot signals \( \sqrt{P_S} \tau(p) \) from individual antennas and feeds back \( \eta = P_S \tilde{\tau}_N \) with \( \tilde{\tau}_N = E[z_{IN}(p)^\dagger z_{IN}(p)]/M \). In other words, the conventional method is identical to the proposed method which approximates \( \tilde{\tau}_{IN} \) by \( \tilde{\tau}_N I \). The terminal A performs the same control procedures to determine the transmit weights \( w_n \) and to predict the SINRs \( \gamma_n^{(p)} \) as in the proposed method.

### C. SINR Prediction

Let us evaluate accuracy of SINR prediction. From simulation results under \( P_S/P_N = 10 \) [dB] and \( (N, M) = (3, 3), (2, 3) \), the conventional method always has SINR prediction error larger than 5 [dB] in the first beamformer \( n = 1 \). This is because the conventional method does not consider the colored interference effect and the predicted SINR becomes far from the actual output SINR. Therefore, it is difficult in practice to predict the SINR without considering the colored interference effect properly.

In the remaining of performance evaluation, we invoke a special assumption that the conventional method uses the same number of transmit beamformers as the proposed method, although the transmit beamforming is different. Table II lists active rate of the \( n \)-th beamformer and SINR prediction errors \( \sigma_n^{\text{ideal}}, \sigma_n^{\text{prop}} \), and \( \sigma_n^{\text{conv}} \) for \( n = 1, 2 \), where \( \sigma_n^{\text{ideal}}, \sigma_n^{\text{prop}} \), and \( \sigma_n^{\text{conv}} \) are the SINR prediction errors in the proposed method with ideal MMSE receiver, in the proposed method with actual MMSE combiner using \( p_1 = 8 \) symbols, and in the conventional method with actual MMSE combiner using \( p_1 = 8 \) symbols, respectively. It is seen that the proposed method with \( p_1 = 8 \) can keep the SINR prediction error within 1.8 [dB] in the presence of phase errors. Therefore, the terminal A can control the number of transmit beamforming properly depending on MIMO channel and interference conditions.

In case of ideal MMSE combiner, the SINR prediction error always becomes less than 1[dB]. It implies that SINR prediction error is greatly enlarged by the terminal B’s weight convergence. Since the weight convergence is a common problem to all types of MIMO systems, the SINR prediction error inherent in the proposed method is considered as small.

In Table II, the second beamformer becomes active with probability about 50% in case of \( (N, M) = (2, 3) \). This is explained as follows. The terminal A uses only the first beamformer, if the interference’s response \( h^{(1)} \) is mostly included in the subspaces of the terminal A’s two antennas. In contrast, the terminal A uses two beamformers, if the interference is out of the terminal A’s subspaces. Based on the same principle, the terminal A mostly uses two and one beamformers in case of \( (N, M) = (3, 3) \) and \( (N, M) = (2, 2) \), respectively.

### D. Received SINR Characteristics

Figure 3 shows cumulative distribution of the terminal B’s received SINR \( \gamma_0 \) in the proposed and conventional methods under \( P_S/P_N = 10 \) [dB], together with the case of ideal transmit and receive weight control. To focus our evaluation on beamformers with high active rates, the results are shown for the first and second beamformers in case of \( (N, M) = (3, 3), (2, 3) \) and only for the first beamformer in case of \( (N, M) = (3, 2), (2, 2) \). In the figure, the proposed method
always achieves higher received SINR than the conventional method. Under $p_1 = 8$ pilot symbols, the case of $M = 3$ has larger performance gap from the ideal case than the case of $M = 2$, due to slower weight convergence at the terminal B. This performance deterioration can be recovered by using more pilot symbols $p_1 = 16$.

V. CONCLUSIONS

We proposed a new pilot-based CSI feedback method, which enables the transmitter to perform efficient transmit beamforming considering the receiver’s interference. Theoretically, the proposed transmit beamforming is one that can maximize channel capacity of MIMO system. From practical point of view, the proposed method enables better SINR prediction and transmission performance than the conventional CSI feedback which does not consider the colored interference effect. Therefore, the proposed pilot-based CSI feedback is expected to be an essential technique in future MIMO systems.

TABLE II

<table>
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<th>${N, M}$</th>
<th>1st beamformer ($n = 1$)</th>
<th>2nd beamformer ($n = 2$)</th>
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REFERENCES