A NOVEL SPACE-FREQUENCY CODING SCHEME FOR SINGLE CARRIER MODULATIONS

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ABSTRACT

Single-Carrier Frequency Division Multiple Access (SC-FDMA) has been adopted as a possible air interface for future wireless networks. It combines most of the advantages of Orthogonal Frequency Division Multiple Access and the low Peak to Average Power Ratio (PAPR) of single carrier (SC) transmission. Existing transmit antenna diversity techniques such as Space-Time Block Coding and Space-Frequency Block Coding are incompatible either with the system constraints or with the SC nature of SC-FDMA. We propose a novel space-frequency flexible coding scheme compatible with SC-FDMA and we prove its good performance both in terms of PAPR and Bit Error Rate (BER) on frequency selective Multiple Input Multiple Output channels.

I. INTRODUCTION

The well known advantages of Orthogonal Frequency Division Multiple Access (OFDMA) \cite{1} make this multi-carrier (MC) technique very attractive for wireless communications. OFDMA has been selected in several recent international standards and is still considered as a strong candidate for future standardization. Almost all current proposals for the air interface of Beyond Third Generation (B3G) and Fourth Generation (4G) cellular systems involve Orthogonal Frequency Division Multiplexing (OFDM), OFDMA or its derivatives \cite{2} such as spread-spectrum MC multiple access (SS-MC-MA).

Compared to time division multiple access (TDMA)-based transmission, FDMA-based transmission offers significant possibilities to increase coverage by concentrating the available transmit power in a fraction of the channel bandwidth. Nevertheless, MC transmission suffers from one main drawback: The high peak-to-average power ratio (PAPR). This causes undesired nonlinear effects when passing through a high power amplifier (HPA), which result in spectral widening and bit error rate (BER) degradation. In order to avoid nonlinear effects, the input signal should lie in the linear region of the HPA. Increased linear dynamic range requirements impose the use of very costly HPAs. Whereas the use of such HPAs can be envisioned in downlink, the problem is much more delicate in the uplink of cellular systems: Indeed, the output power of user terminals is strictly limited and must be efficiently utilized in order to optimize the coverage and save battery life, while not neglecting the low-cost constraints. Many PAPR reduction techniques have been developed, but they do not always yield significant gains in practical applications \cite{3}. In contrast to MC-FDMA, SC-FDMA seems best suited for the uplink, as it combines the low-PAPR characteristics of SC transmission with the advantages of FDMA. Several methods of generating SC-FDMA signals exist. The original one, called Interleaved FDMA (IFDMA) \cite{4}, relies on time-domain processing only. Another approach is to generate a SC-FDMA waveform using frequency domain processing. The result is a precoded OFDMA transmission where the precoding matrix is chosen to be a Direct Fourier Transform (DFT). This DFT-precoded OFDMA (also called DFT-Spread OFDM) has been adopted for the uplink of future wireless systems by the 3GPP (Third Generation Partnership Project), which focuses on the Long Term Evolution (LTE) of UMTS (Universal Mobile Terrestrial Systems). As for OFDM, robustness to cellular interference can be achieved with SC-FDMA by coordinating resource allocation between adjacent cells. Moreover, multiplexing uplink transmission with different rates is possible, as a different number of subcarriers and a different modulation and coding scheme can be assigned to each user. Nevertheless, in the case of a frequency selective channel, SC-FDMA loses some orthogonality with respect to OFDMA: Interference may occur within the elements of each data block, as a result of spreading the symbols over a set of subcarriers.

The ever increasing demand for high throughput, good spectral efficiency and improved performance impose the use of multiple antennas both at the base station and at the terminals. According to the transmission environment, multiple transmit and receive antennas may be used to increase diversity and improve BER performance or increase the transmitted data rate through spatial multiplexing, and/or reduce interference from other users \cite{5,6}. Whereas a terminal with good propagation conditions may employ its transmit antennas for spatial multiplexing, a terminal located at the cell-edge may take more benefit from multiple antennas to increase diversity and coverage by precoding in the space dimension. The existing precoding-based transmit antenna diversity techniques are unfortunately incompatible either with some framing constraints or with low PAPR constraints, for reasons that will be detailed in the sequel. In this paper, we propose a novel space-frequency precoding scheme, which does not alter the PAPR of SC-FDMA.

This paper is structured as follows: Section II describes the system model. Section III presents existing precoding-based transmit diversity techniques and describes our proposed scheme. Simulation results are given in section IV. Finally, conclusions are presented in section V.

II. SYSTEM MODEL

The concept of SC-FDMA was firstly introduced by a time-domain implementation called IFDMA \cite{3}. The input data stream is split into symbol blocks $\mathbf{x}^{(i)}$:

$$\mathbf{x}^{(i)} = [x_0^{(i)}, x_1^{(i)}, \ldots, x_{M-1}^{(i)}]$$  \hspace{1cm} (1)
of duration \( T = MT \), where \( T_s \) is the sampling period. Each block is compressed and repeated \( K \) times. The blocks:

\[
\tilde{x}^{(i)} = \left[ x^{(i)}, x^{(i)}, \ldots, x^{(i)} \right]_{K \text{ times}}
\]

of the same duration \( T \) contain \( N = KM \) samples corresponding to a sampling period \( T_s = T/K \). A user-dependent phase is also defined. For user \( q \), the phase sequence is given by:

\[
\Phi_{q}^{(i)} = q \frac{2\pi}{N}, \quad (q = 0 \ldots N - 1).
\]

As a result, each block \( \tilde{x}^{(i)} \) given by Eq. (2) is multiplied with a user-specific phase ramp, resulting in blocks \( y^{(i)} \):

\[
y^{(i)} = \left[ x_0^{(i)} e^{-j\Phi_{q}^{(i)}}, x_1^{(i)} e^{-j\Phi_{q}^{(i)}}, \ldots, x_{N-1}^{(i)} e^{-j(N-1)\Phi_{q}^{(i)}} \right].
\]

This manipulation has a direct interpretation in the frequency domain. The spectrum of the compressed and \( K \)-times repeated signal has the same shape as the original signal, with the difference that it presents exactly \( K \) zero points in-between two data subcarriers. The user-specific phase ramp (corresponding to a certain frequency shift) allows to easily interleave in the frequency domain the signals corresponding to different users.

Alternatively, SC-FDMA can be easily generated using frequency domain processing: DFT-precoded OFDMA with DFT outputs mapped on regularly distributed subcarriers would generate the same waveform, the two implementations being mathematically equivalent. In this case, the SC property is retrieved via the DFT precoding operation. Moreover, this kind of implementation is not confined to a distributed scenario: DFT outputs may be mapped on localized subcarriers, which opens the door to channel-dependent subcarrier allocation. Because of this larger flexibility with respect to IFDMA, frequency domain implementation of SC-FDMA will be focused on in the sequel.

Most of the various classes of future generation mobile terminals will employ multiple transmit and receive antennas to satisfy constraints of coverage and throughput demands. The way these antennas should be used strongly depends on the propagation conditions. It is thus the responsibility of the link adaptation mechanism to jointly select the modulation alphabet, the coding rate and the antenna scheme according to the variations of these conditions.

Transmit diversity is an effective means to improve performance. The most popular transmit diversity technique has been proposed by Alamouti [7]. Even though it does not increase the throughput, this ingenious yet simple scheme has several advantages that render it attractive: Coding and decoding are very simple while providing optimum performance, and diversity is doubled, without any requirement on channel knowledge at the transmitter side. It is effective in all applications where system capacity is limited by multipath fading. It is naturally well combined with OFDM or OFDMA-like techniques.

Fig.1 presents the block diagram of an uplink SC-FDMA transmitter which is allocated a subset of \( M \) out of \( N \) subcarriers and is equipped with two transmit antennas, making use of Alamouti transmit diversity. At time \( i \), data block vector \( x^{(i)} \), which is composed of \( M \) modulation symbols \( x_k^{(i)} (k=0 \ldots M-1) \), e.g., Quadrature Phase Shift Keying (QPSK) symbols, is DFT-precoded by means of a \( M \)-sized DFT. \( M \)-sized vectors \( s^{(i)} \) thus obtained are Alamouti precoded, resulting in \( M \)-sized vectors \( s_1^{(i)} \) and \( s_2^{(i)} \). These vectors are then mapped on \( M \) out of \( N \) inputs of the inverse DFT according to the subcarrier mapping strategy in order to be transmitted on antennas \( Tx_1 \) and \( Tx_2 \), respectively. To combat the effect of the frequency selective channel, a cyclic prefix (CP) is inserted in front of each \( N \)-sized block thus obtained.

Note here that Alamouti precoding is processed between the frequency components of single-carrier signals, i.e., on vectors \( s^{(i)} \) obtained after DFT. As an alternative strategy, Alamouti precoding could be applied between the time components \( s_k^{(i)} \) of single-carrier vector \( x^{(i)} \) (before DFT). The latter approach would require two DFT modules (one on each antenna branch), leading to increased complexity and power consumption in user terminals. Moreover, the receiver of such a system would also suffer from a higher complexity: Decoding can no longer be performed on pairs of subcarriers but would involve all of the subcarriers.

### III. SPACE-TIME AND SPACE-FREQUENCY BLOCK CODES

There are several ways of applying an Alamouti precoding on the frequency components of a SC-FDMA signal. However, three constraints have to be considered. First, precoding should better be applied on pairs of frequency components \( (s_k^{(i)}, s_{k+M}^{(i)}) \) that experience similar channel realizations so as to get optimum performance with a simple maximum ratio combiner (MRC) [7]. Second, precoding should not impact the low PAPR property of SC-FDMA signals, whatever the number of sub-carriers allocated to a terminal. Third, precoding should be compatible with practical system constraints such as frame length.

![General SC-FDMA transmitter’s block diagram](image)

Figure 1: General SC-FDMA transmitter’s block diagram \((M \text{ out of } N \text{ allocated subcarriers, } 2 \text{ transmit antennas})\).
A. Space-Time Block Coding (STBC)

The most straightforward approach for applying Alamouti coding is known as STBC: it performs a precoding operation between two components \( s_{(2i)} \) and \( s_{(2i+1)} \) onto two antennas over two successive time intervals. Precisely, Alamouti precoding is performed between \( k \)-th frequency component \( s_{(2i)} \) of DFT output vector \( \mathbf{s} \) at time 2 and the same \( k \)-th frequency component \( s_{(2i+1)} \) of successive output vector \( \mathbf{s} \) at time 2+1, as defined in Table 1. The precoded components are then mapped onto two consecutive SC-FDMA blocks.

Since the same manipulation is performed for all occupied carriers, the frequency structure of the signal is not impacted and we can consider that Alamouti precoding is performed at block level. Moreover, because complex conjugate operations and/or sign changes at this level do not break the low PAPR feature, the signals sent on the two transmit antennas are both single-carrier type signals.

Nevertheless, STBC has several weaknesses. This type of precoding assumes that SC-FDMA blocks are coded by pairs, and thus all frames would be constrained to have an even integer. Once the associations \( k_1 \) and \( k_2 \) are established, \( k_1 \) is even and type 1 variant if \( k_1 \) is even and type 2 variant if \( k_1 \) is odd, as in the example in Fig. 2.

B. Classical Space Frequency Block Coding (SFBC)

As an alternative, Alamouti precoding may be applied in the frequency dimension. SFBC encodes two components \( s_{i1} \) and \( s_{i2} \) onto two antennas over two different frequencies. Classically, these two frequencies are chosen to correspond to adjacent subcarriers, in order to limit the variations of the channel. Precisely, precoding is performed between \( k \)-th frequency component \( s_{(i)} \) of DFT output vector \( \mathbf{s} \) at time 1 and the successive \( k+1 \)-th frequency component \( s_{(i+1)} \) belonging to the same DFT output vector \( \mathbf{s} \), as shown in Table 2. Since SFBC is applied within each DFT output vector, it can be applied independently of the number of SC-FDMA blocks, control and pilot signals per frame. Of course, the size of vector \( \mathbf{s} \) must be an even number, but in practical systems this condition is usually already verified.

With respect to STBC, SFBC is more sensitive to large delay spreads [9] since channel realisations on adjacent subcarriers may differ. Nevertheless, the main problem in our case of interest arises from the fact that SFBC consists of manipulations that change the frequency structure of the signal. The signal transmitted on Tx1 is a single-carrier type signal, thanks to the specific variant of the Alamouti scheme that we have used. Since frequency inversions between successive subcarriers are performed, the single-carrier property of the signal on transmit-antenna Tx2 is altered and the advantage of the low PAPR of SC-FDMA is lost as shown in section IV.

C. Single-Carrier SFBC (SC-SFBC)

In this paper, we propose a new Alamouti-based SFBC precoding scheme, called SC-SFBC since it conserves the single carrier property of SC-FDMA on both transmit antennas. Our idea consists of using simultaneously two different types of Alamouti precoding in the frequency domain, as indicated in Table 3. Both types guarantee that the original SC signal is sent on the first transmit antenna. The two frequency inversions involved in the Alamouti precoding do not necessarily correspond to successively used subcarriers any longer, but to carriers \( k_1 \) and \( k_2 = (p - 1) \mod M \) where \( p \) is an even integer. Once the associations \( k_1 \leftrightarrow k_2 \) are established, we use type 1 variant if \( k_1 \) is even and type 2 variant if \( k_1 \) is odd, as in the example in Fig. 2.

As shown in Table 3, the SFBC precoding types are:

\[
\mathbf{s}_{k} = (-1)^{k+1} \mathbf{s}_{(p-1) \mod M}^* \quad (k = 0...M - 1)
\] (5)

which is equivalent to sending on the second transmit antenna.
different channel realizations on subcarriers and affected by the same frequency selective channel. Therefore, SC-SFBC is expected to be more sensitive than SC transmission (i.e., STBC), while the classical SFBC mappings have shown the same good behaviour for our proposed scheme, who manages to keep the same signal distribution as the equivalent SC transmission.

In the design and analysis of Alamouti schemes, it is generally assumed that the frequency components being encoded together should be mapped so as to benefit from the same channel realization. In SFBC scenarios, channel realizations may differ significantly, especially when these subcarriers are placed rather far apart from each other in the spectrum. In order to evaluate the degradation caused by SFBC precoding, we evaluated the system performance on a BRAN E multipath channel [10]. We did not consider any spatial correlation among the different transmit/receive antennas, assuming sufficient antenna spacing and/or large angular spread.

In the uplink, localized allocations of subcarriers are usually considered, since channel estimation can be much more easily performed than in the distributed case. One or several resource units might be allocated to the same user, i.e., $M$ is a multiple of 12. Fig. 4 presents simulation results in the case of 60 localized subcarriers (5 resource units) allocated to the same user. 2 or 4 antennas are used at the base-station. Although our proposed SC-SFBC precodes pairs

IV. SIMULATION RESULTS

The simulated system employs SC-FDMA with $N = 512$ subcarriers, among which 300 are modulated data carriers (the remaining 212 are guard carriers). The 300 active subcarriers are split into 25 resource units of 12 subcarriers. To fit a channelization bandwidth of 5 MHz, the sampling frequency is 7.68 MHz. The signal constellation is QPSK. We employ a $(753, 531)$ convolutional code with rate $1/2$. Data is scrambled before coding and interleaved prior to QPSK mapping. Groups of 8 SC-FDMA blocks are encoded together and affected by the same frequency selective channel. Let $y$ be a suite of digital samples. We define the Complementary Cumulative Distribution Function (CCDF) of the Instantaneous Normalized Power (INP) of $y$:

$$\text{CCDF}_{\text{INP}(y)}(\gamma^2) = \Pr \left( \frac{|y|}{E[|y|^2]} > \gamma^2 \right). \quad (7)$$

as being the probability that the INP of a sample $y_i$ exceeds a certain clipping level $\gamma^2$.

The CCDF of INP takes into account all the samples of the signal and gives a more accurate signal description than the largely employed CCDF of PAPR, which considers only the largest peak of each block of samples. Fig. 3 presents the CCDF of INP of the signal $y_{2,n}$ (at the input of the HPA before Tx2), and corresponding to the three strategies described above (STBC, SFBC and SC-SFBC precoding). An oversampling factor of 4 is considered, since it is known to give a good approximation for the PAPR of the corresponding continuous-time signal. We can see that using the proposed SC-SFBC mapping yields the same signal distribution as in SC transmission (i.e., STBC), while the classical SFBC suffers a loss of 1.1 dB at clipping probability of $10^{-4}$. Simulations performed using 16QAM and 64QAM signal mappings have shown the same good behaviour for our proposed scheme, who manages to keep the same signal distribution as the equivalent SC transmission.

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![Figure 3: CCDF of INP, QPSK transmission.](image-url)
of frequency components that may experience quite different channel realizations, it doesn’t bring any significant performance loss. The decoding strategy used here is MRC. Alamouti decoding must be followed by per-carrier equalization. Slightly larger degradation is obtained when a larger number of subcarriers is allocated to the same user, but this is rarely the case in practical FDMA configurations, especially for terminals located at the cell-edge.

Moreover, in the case where degradation is expected, the use of a slightly more complex decoding strategy can be taken into consideration in the base station. A Minimum Mean Square Error (MMSE) decoder, which reduces the intersymbol interference due to channel selectivity in an Alamouti precoded pair, is a natural choice. Decoding is performed by pairs of frequency components coded together: matrices of order 2 need to be inverted, which can be done with relatively low computational complexity. Fig. 5 presents the results obtained with MMSE decoder in the case of 12 distributed subcarriers (one resource unit) allocated to the same user. This is only presented here as a worst case scenario for SC-SFBC: The subcarriers involved in Alamouti coding are distant (separated by at most 124 subcarriers) and the channel is stationary, which favors STBC. At a target BER of $10^{-4}$, the classical SFBC loses 0.3 dB with respect to the STBC scheme when 2 receive antennas are used. As expected, our proposed SC-SFBC suffers slightly larger loss, i.e., 0.7 dB. However, when 4 receive antennas are used, the degradation is of only 0.1 dB and 0.2 dB respectively.

V. CONCLUSIONS

Existing transmit diversity schemes suffer from several characteristics which render them either difficult to use from a system point of view, or incompatible with SC-FDMA transmission. STBC doubles the granularity and imposes a heavy constraint: All frames must contain an even number of SC-FDMA blocks, which cannot always be assured. Classical SFBC breaks the key advantage of SC-FDMA transmission: Its single carrier property.

We have proposed a new SFBC coding scheme compatible with SC-FDMA transmission and any frame size. It has the advantage of keeping the low PAPR of SC-FDMA and it has negligible performance degradation in various practical scenarios.

Future work needs to be done in order to evaluate the performance of this new scheme on channels with time selectivity and spatial correlation among antennas.

REFERENCES


